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## EIGHTH MONTHLY PROGRESS REPORT, 1-31 Jan. 1964 (NASA CR-53397) OTS

(NASA Contract NAS 2-1460)

Effort during this reporting period, January 1 - 31, 1964, includes dividing the combined program up into four main programs, continuing check-out of the various routines, and finalizing the boundary layer routines.

Combined Program

As mentioned in the Seventh Monthly Progress Report, difficulties were encountered in the overlay system of Fortran IV. Fortran IV can handle approximately 200 subroutines, and the present program contains approximately 480 routines. It has thus been established that the present program is too large for one complete program. Consequently, the combined program is presently being divided into four main programs where these programs may be loaded on one tape and run consecutively. The four programs are as follows:

1. External Flow - This program computes the complete viscous-inviscid flow field up to the cowl shock wave.
2. Blunt Cowl-Lip - This program computes the flow field around the blunted cowl-lip using the Ames blunt body program and the boundary layer routine.
3. Internal Flow with No Shock Intersections - This program computes the flow field downstream of the cowl-lip shock to any shock intersections.
4. Internal Flow with Shock Intersections - This program computes the flow field downstream of the first shock intersection to the end of the inlet.

OTS PRICE

XEROX $ 1.60 ph

MICROFILM $ 0.80 mf.

Most of the effort during this reporting period has been devoted to breaking the original program up into these four programs.

Ames Test Case

The test case supplied by Ames has been delayed due to the fact that the routine that computes the intersection of the cowl lip shock and bow shock waves has not been completely integrated with the overall program. The above is also true of the vortex sheet routine and the routine for the computation of the flow field, both of which result from the intersection of shocks of opposite family. These routines are currently being integrated with the program. The routine for the intersection of shock waves of the opposite family is discussed in the following section following which is a discussion of the stagnation point boundary layer routine.

It is to be noted that difficulties with the Fortran IV System have caused a considerable delay in this program. We are currently reviewing the status of the complete program to determine whether the scheduled contract completion date will be met.

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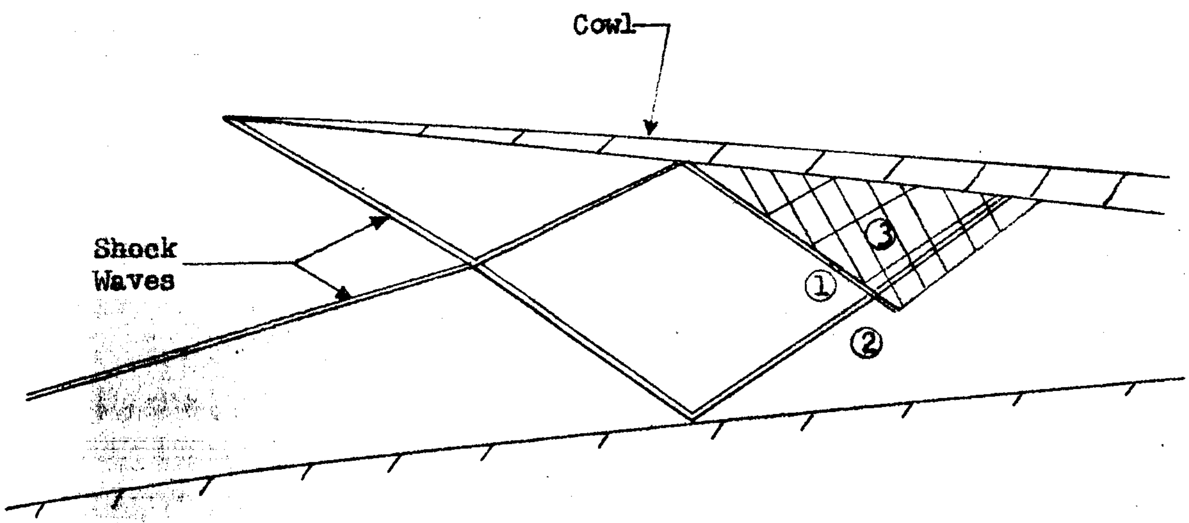
Intersection of Shock Waves of Opposite Family

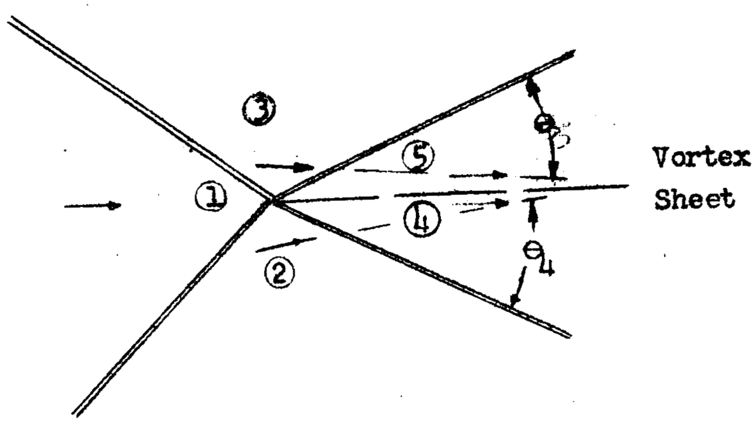
The solution for the intersection of shocks of the opposite family is relatively straightforward when the fluid involved is a perfect gas. The shock polar equations are solved in conjunction with the boundary conditions of the vortex sheet. The existence of more than one solution creates the only difficulty involved as the computer must select the solution where both reflected oblique shocks are of the weak variety. Experiment indicates that this is the only stable situation which will exist physically. It is also possible to have "Mach reflection" in which case no analytic solutions will exist. The treatment of shock intersections for a real gas is somewhat more involved, however, and an outline of the solution is given below. The numbering of regions is as shown in the illustration for the general case where the conditions upstream of the intersection are not free stream.



1. Regions 1 and 2 are completely computed. The down shock is calculated and as each shock point is determined, it is tested for intersection with the shock separating regions 1 and 2. Once the intersection point is established, the region 3 properties are computed up to and including the first family characteristic passing through the downshock point immediately downstream of the intersection. This data is then stored. The value of $\theta$, the shock angle is determined at the intersection point by linear interpolation.

2. $P_1$, $S_1$ and $\delta_1$, (stream angle) are found at the intersection point from the region 1 curve fit. The fluid properties in regions 2 and 3 are computed from the shock point routine, with $\theta$ known in this case.

3.




The boundary conditions on the vortex sheet are that $P_4 = P_5$ and $\delta_4 = \delta_5$. As a first guess for $\delta$, assume that the turning strength of each shock is the same after reflection as before intersection.

$$\delta_4^{(1)} = \delta_5^{(1)} = \delta_2 + (\delta_3 - \delta_1)$$

4. For a first guess as to the shock angle of the shock separating regions 3 and 5, use 0.95 times the perfect gas $\theta$ calculated from the following equation:

$$\sin^6 \theta_5 - \left[ \frac{M_3^2 + 2}{M_3^2} + \gamma_3 \sin^2 \left( \delta_5^{(1)} - \delta_3 \right) \right] \sin^4 \theta_5$$

$$+ \left\{ \frac{2M_3^2 + 1}{M_3^4} + \left[ \frac{(\gamma_3 + 1)^2}{4} + \frac{(\gamma_3 - 1)}{M_3^2} \right] \sin^2 \left( \delta_5^{(1)} - \delta_3 \right) \right\} \sin^2 \theta_5 -$$

$$- \frac{\cos^2 \left( \delta_5^{(1)} - \delta_3 \right)}{M_3^4} = 0$$

$\theta_5$ in this equation is relative to $\delta_3$. Use the intermediate value of the 3 solutions for $\theta_5$.

5. The equations for conservation of mass, momentum and energy are applied at the shock wave to obtain 2 different values of $h_5$. When the two agree, the correct value of $\theta_5$ has been determined.

6. If the first guess is not correct, use a Newton-Raphson iteration procedure to converge on proper value of $\theta$. Use $\theta_5^{(2)} = 1.01\theta_5^{(1)}$, then compute a better estimate from equation:

$$\theta_5^{(3)} = \theta_5^{(2)} - \frac{(h_5' - h_5)^{(2)} (\theta_5^{(2)} - \theta_5^{(1)})}{(h_5' - h_5)^{(2)} - (h_5' - h_5)^{(1)}}$$

i.e.: $$\theta_5^{(3)} = \theta_5^{(2)} - \frac{d\theta}{d(h_5' - h_5)} (h_5' - h_5)$$

Continue above procedure until a value of $\theta_5$ is found for which

$$\left| h_5 - h_5' \right| \leq h_5 \text{ x convergence factor}$$

Obtain the remaining fluid properties in region 5 from R-gas program, entering with $P_5$ and $\rho_5$.

7. Repeat steps 4 through 6 to find $\theta_4$ and the properties in region 4.

8. So far, values of $\theta_5$ and $\theta_4$ have been determined such that the boundary condition $\delta_4 = \delta_5$ is satisfied. However, a value of $\delta_{4,5}$ must now be found for which $P_4 = P_5$. Compare the final values of $P_4$ and $P_5$. Designate the difference between these two quantities by $(P_5 - P_4)^{(1)}$, since they are based on the first estimate of $\delta_4$. The second estimate for $\delta_4$ will be one of the following:

$$\left.\begin{aligned} \delta_4^{(2)} &= \delta_5^{(2)} - 0.99\ \delta_5^{(1)} \quad \text{for } \delta_5^{(1)} > 0^\circ \\ \delta_4^{(2)} &= \delta_5^{(2)} = 1.01\ \delta_5^{(1)} \quad \text{for } \delta_5^{(1)} < 0^\circ \end{aligned}\right\} \text{if } P_5^{(1)} \quad P_4^{(1)}$$

$$\left.\begin{aligned} \delta_4^{(2)} &= \delta_5^{(2)} = 1.01\ \delta_5^{(1)} \quad \text{for } \delta_5^{(1)} > 0^\circ \\ \delta_4^{(2)} &= \delta_5^{(2)} = 0.99\ \delta_5^{(1)} \quad \text{for } \delta_5^{(1)} < 0^\circ \end{aligned}\right\} \text{if } P_5^{(1)} \quad P_4^{(1)}$$

9. Repeat steps 4 through 7, finally obtaining $\theta_4$ and $\theta_5$ which will produce flow direction $\delta_4^{(2)} = \delta_5^{(2)}$. Once again compare pressures $P_4$ and $P_5$. Note the difference and designate by $(P_5 - P_4)^{(2)}$.

10. Compute a better value of $\delta_4 = \delta_5$ from equation:

$$\delta_4^{(3)} = \delta_5^{(3)} = \delta_{4,5}^{(2)} - \frac{(P_5 - P_4)^{(2)}\ (\delta_{4,5}^{(2)} - \delta_{4,5}^{(1)})}{\left[(P_5 - P_4)^{(2)} - (P_5 - P_4)^{(1)}\right]}$$

11. Repeat steps 4 through 7 for $\delta_4^{(3)}$. Compute improved value of $\delta_{4,5}$. Continue this procedure until a value of $\delta_{4,5}$ is found for which

$$|P_5 - P_4| \leq P_5 \times \text{convergence factor.}$$

12. At no time in the iterative procedure shall a value of $P_5$ or $P_4$ be used which exceeds the pressure computed below:

$$\left(\frac{P_5}{P_3}\right)_{max} = \frac{-(M_3^2-2)(\gamma_3+1)+\sqrt{(\gamma_3+1)\left[(\gamma_3+1)M_3^4+8(\gamma_3-1)M_3^2+16\right]}}{2(\gamma_3+1)}$$

$$\left(\frac{P_4}{P_2}\right)_{max} = \frac{-(M_2^2-2)(\gamma_2+1)+\sqrt{(\gamma_2+1)\left[(\gamma_2+1)M_2^4+8(\gamma_2-1)M_2^2+16\right]}}{2(\gamma_2+1)}$$

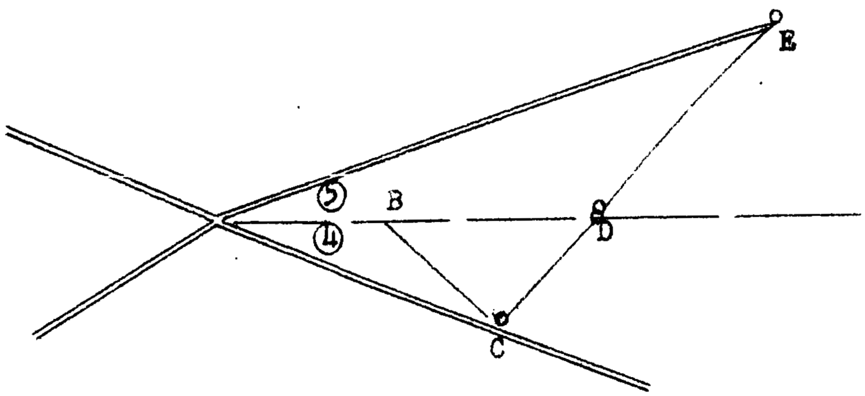
The above expressions are for the pressure at the detachment value of $\delta$. If the static pressure is below the detachment pressure, then the weak shock solution is assured.

13. Once the final values of $\Theta_4$, and $\delta_5$ have been determined, the properties of the air in regions 4 and 5 are obtained from the R-gas program.

14. The scheme for computing the flowfield downstream of the intersection will now be described briefly. Assume a point "B" on the vortex sheet to have the same properties as at the intersection. There will be two sets of properties at this point; one set corresponding to the flowfield above the vortex sheet and one set below. Let X of this point be 1.0001 X of the intersection point. Find y from equation:

$$(y_B - y_A) = (X_B - X_A)\tan\delta_{4,5}$$

15. Compute $\Theta$ and the fluid properties at point C using down shock routine. Assume $\delta_D = \delta_B$ and locate point D as the intersection of the vortex sheet and the first family characteristic through point C.




16. The properties at point D below the vortex sheet are found from the characteristics equation and the known entropy which remains constant along the lower surface of the vortex sheet.

17. Once the properties at the lower surface at D are known, the upper portion may also be computed. The pressure is the same on either side of the vortex sheet and the entropy will be equal to $S_5$ at all points along the upper surface.

18. The shock point E may now be computed from the up shock routine.

19. The general procedure for the rest of the flowfield is to compute along first family rays, beginning with the downshock, computing across the vortex sheet and finishing at the up shock. A test is incorporated to determine when the vortex sheet is being approached and the computer transfers to the vortex sheet routine at this time. The vortex routine utilizes an iterative procedure whereby the boundary conditions of the vortex sheet are satisfied along with the characteristics equations. The procedure is continued until either shock is reflected from the centerbody or cowl at which time the procedure is modified as required.

## Stagnation Point Boundary Layer Routine

The boundary layer solution in the stagnation region of a blunt body was discussed in the Fifth Monthly Progress Report. At the stagnation point, however, special care must be taken in some of the parameters. This is due to the coordinate $x$, the local velocity $U_e$, and the transformation variable $\xi$ are all equal to zero at the stagnation point. At the stagnation point the velocity gradient is defined, Reference 1,

(1) $$\frac{dU_e}{dx} = \frac{U_e}{x} \qquad \text{and by definition}$$

(2) $$\beta = \frac{2\xi}{U_e}\,\frac{dU_e}{d\,x}$$

$$= \frac{2\xi}{U_e}\,\frac{dU_e}{dx}\,\frac{dx}{d\xi}. \qquad \text{But}$$

(3) $$\xi = \frac{1}{\mu_w^{\,2}}\int_0^x \rho_w \mu_w U_e r^{2j}\,dx.$$

Differentiating equation (3) with respect to x and substituting in equation (2) gives

(4) $$\beta = \frac{2\xi}{U_e}\,\frac{dU_e}{dx}\,\frac{\mu_w}{\rho_w U_e r^{2j}}$$

Then the parameter $\sqrt{2\xi}$ may be found from equation (4) giving

(5) $$\sqrt{2\xi} = \left[\frac{\beta \rho_w U_e^{\,2} r^{2j}}{\frac{dU_e}{dx}\mu_w}\right]^{\frac{1}{2}} \qquad \text{and}$$

(6) $$\frac{\rho_w r^j U_e}{\sqrt{2\xi}} = \left[\frac{\rho_w \mu_w \frac{dU_e}{dx}}{\beta}\right]^{\frac{1}{2}}$$

The stagnation point velocity gradient $\frac{dU_e}{dx}$ is found from Newtonian flow, Reference 2,

(7)
$$\frac{dU_e}{dx} = \frac{1}{R_B}\left[2(P_e - P_\infty)/\rho_e\right]^{\frac{1}{2}}$$

Equation (6) becomes

(8)
$$\frac{\rho_w r^j U_e}{\sqrt{2\xi}} = \left\{\frac{\rho_w}{\beta} \quad \frac{\mu_w}{R}\left[\frac{2(P_e - P_\infty)}{\rho_e}\right]^{\frac{1}{2}}\right\}^{\frac{1}{2}}$$

Equation (8) may be substituted in the heat flux and displacement thickness terms to determine the parameters at the stagnation point. For frozen flow the heat transfer parameter, given in equation 17 of the Fifth Monthly Progress Report, becomes

(9)
$$\frac{N_u}{P_r\sqrt{R_w}} = \frac{(778.2/3600)\ q_w}{H_e^{*}\left\{\frac{\rho_w \mu_w}{R_B}\left[2(P_e - P_\infty)/\rho_e\right]^{\frac{1}{2}}\right\}^{\frac{1}{2}}}$$

where the heat flux

(10)
$$q_w\Big|_{frozen} = \frac{3600\ H_e}{778.2\ P_r}\left\{\frac{\rho_w}{\beta}\ \frac{\mu_w}{R_B}\left[\frac{2(P_e - P_\infty)}{\rho_e}\right]^{\frac{1}{2}}\right\}^{\frac{1}{2}}$$
$$\left\{\zeta_\eta\ (o) + \alpha_e\ \frac{h_1^{o}}{H_e}\left[(L_e - 1)\quad Z_{A_\eta}(o)\right]\right\}_w .$$

The pressure gradient parameter $\beta$ is obtained from the following table

| $\beta$ | j | Flow |
|---|---|---|
| 1 | o | Stagnation Point Two-Dimensional |
| $\frac{1}{2}$ | 1 | Stagnation Point Axisymmetric |

The displacement thickness is written

(11) $$\delta^{*} = \frac{\sqrt{2\,\xi\,\mu_w}}{\rho_e\, U_e r^j} \int_0^{\eta_t} \left( \frac{\rho_e}{\rho} - f_\eta \right) d\eta$$

and at the stagnation point

(12) $$\delta^{*} = \frac{\sqrt{\rho_w \mu_w \beta R_B}}{\rho_e} \left[ \frac{2(P_e - P_\infty)}{\rho_e} \right]^{-\frac{1}{4}} \int_0^{\eta_t} \left( \frac{\rho_e}{\rho} - f_\eta \right) d\eta ,$$

Similarly, the momentum thickness $\theta$ becomes

(13) $$\theta = \frac{\sqrt{\rho_w \mu_w \beta R_B}}{\rho_e} \left[ \frac{2(P_e - P_\infty)}{\rho_e} \right]^{-\frac{1}{4}} \int_0^{\eta_t} (1 - f_\eta) f_\eta \, d\eta ,$$

For equilibrium flow the heat transfer parameter, equation (9) is identical, but the heat flux $q_w$ becomes

(14) $$q_w \Big|_{equil.} = \frac{3600\ H_e}{778.2\ P_r} \left\{ \frac{\rho_w}{\beta} \frac{\mu_w}{R_B} \left[ \frac{2(P_e - P_\infty)}{\rho_e} \right]^{\frac{1}{2}} \right\}^{\frac{1}{2}} \left\{ g_\eta(0) \right\}$$

The remaining parameters, $\delta^{*}$ and $\theta$, are identical for both frozen and equilibrium flow.

The turbulent boundary layer program is presently being finalized, and the details of the program will be given in the near future.

## NOMENCLATURE

| | | |
|---|---|---|
| $c_p$ | - | specific heat |
| $D_{12}$ | - | bimolecular diffusion coefficient |
| $f$ | - | velocity ratio, $u/u_e$ |
| $h_1^o$ | - | enthalpy of formation of species |
| $h$ | - | static enthalpy |
| $h_e$ | - | reference enthalpy |
| $H$ | - | total enthalpy |
| $j$ | - | exponent of body radius r |
| $k$ | - | thermal conductivity |
| Le | - | Lewis number |
| Nu | - | Nusselt number |
| $P_t$ | - | total pressure |
| $P$ | - | static pressure |
| Pr | - | Prandtl number $\frac{\mu c_p}{k}$ |
| R | - | Reynolds number |
| $r$ | - | radius of body at revolution |
| $T$ | - | absolute temperature |
| $u, v,$ | - | velocity components in x and y direction respectively |
| $x$ | - | distance along body surface |
| $y$ | - | distance normal to body surface |
| $Z_A$ | - | mass fraction ratio $\alpha/\alpha_e$ |
| $\alpha$ | - | mass fraction of atoms |
| $\beta$ | - | pressure gradient parameter |
| $\delta$ | - | total enthalpy ratio, $H/H_e$ |
| $\mu$ | - | viscosity coefficient |

$\xi, \eta$ - similarity variables

$\eta_t$ - $\eta$ @ the edge of the boundary layer

$\rho$ - mass density

$\tau$ - shear stress

$\varphi$ - density viscosity product ratio, $\rho\mu/\rho_w\mu_w$

$\theta$ - angle between tangent of body and centerline

Subscripts

E - evaluated at reference enthalpy $H_e$ and local pressure

e - local value external to boundary layer

w - evaluated at wall

$x, y, \eta, \xi$ - derivative with respect to $x$, $y$, $\eta$, $\xi$.

## REFERENCES

1. Cohen, N. B., "Boundary Layer Similar Solutions and Correlation Equations for Laminar Heat Transfer Distribution in Equilibrium Air at Velocity up to 41,100 ft/sec," NASA TR R-118, 1961.

2. Fay, J. A. and Riddell, F. R., "Theory of Stagnation Point Heat Transfer in Dissociated Air," Journal of Aeronautical Sciences, pp. 73-85. February 1958.